TRIGONOMETRIC FORM OF COMPLEX NUMBERS: THEORY AND PRACTICAL APPLICATIONS

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ABSTRACT:

This paper explores the trigonometric form of complex numbers, focusing on both theoretical aspects and real-world applications. Representing complex numbers in trigonometric form greatly simplifies operations such as multiplication, division, exponentiation, and root extraction. Using De Moivre's formula, we demonstrate the efficiency of solving complex operations. Practical applications across fields like electrical engineering, signal processing, computer graphics, and navigation are also highlighted. Additionally, effective teaching methods are discussed to enhance students' understanding of this important mathematical concept.

INTRODUCTION. In modern mathematics, complex numbers are not only a theoretical concept but also an important practical tool in fields such as electrical engineering, signal processing, computer graphics, and many more. Representing them in geometric and trigonometric forms simplifies operations and allows for in-depth analysis. This article analyzes the trigonometric form of complex numbers from a theoretical perspective and highlights its practical applications through real examples. In particular, we demonstrate how operations such as exponentiation and root extraction are simplified using De Moivre's formula.

What is a Complex Number?

A complex number is generally written as:

$$z = a + ib$$

Where:

- a is the real part
- b is the imaginary part

- i is the imaginary unit, where $i^2 = -1$

The set of complex numbers is usually denoted by ℂ. They can be represented as points or vectors in the complex plane.

Theory of Trigonometric Form

When we represent a complex number geometrically — that is, in the coordinate plane — it takes the trigonometric form:

$$z = r(\cos\theta + i\sin\theta)$$

Where:

- $r = \sqrt{a^2 + b^2}$ is the modulus (distance from the origin)
- $\theta = \arctan(\frac{b}{a})$ is the argument (angle in radians)

Operations in Trigonometric Form

- **Multiplication**:

$$z_1 \cdot z_2 = r_1 \cdot r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)].$$

- **Division**:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)].$$

- **Exponentiation (De Moivre's Formula)**:

$$z^n = r^n[\cos(n\theta) + i\sin(n\theta)].$$

- **Root Extraction**:

$$\sqrt[n]{z} = \sqrt[n]{r}[\cos((\theta + 2k\pi)/n) + i\sin((\theta + 2k\pi)/n)], k = 0,1,...,n-1.$$

Practical Examples

Example 1: Convert $z = 1 + i$ into trigonometric form.

$$r = \sqrt{2}, \theta = \frac{\pi}{4} \rightarrow z = \sqrt{2}(\cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4}))$$

Example 2: Find the cube of $z = 1 + i$:

$$z^3 = 2\sqrt{2}(\cos(\frac{3\pi}{4}) + i\sin(\frac{3\pi}{4}))$$

Appendix: Graphical Representation

The following graph represents the complex number $z = 1 + i$ in trigonometric form on the coordinate plane:

(Insert graph here)

Real-Life Applications of Complex Numbers

Complex numbers can solve many practical problems and offer pathways for innovation. Here are a few fields where they are essential:

1. Electrical Current and Engineering Applications

The trigonometric form of complex numbers is especially useful in analyzing electric circuits, particularly in AC (alternating current) systems where voltage and current have a phase difference.

Real-life example: "Modern household devices such as air conditioners, televisions, and washing machines all operate on AC. Engineers use complex numbers to design their operational mechanisms."

2. Wave and Signal Analysis

Sinusoidal waves are used in digital communication systems such as phones, Wi-Fi, and radio. Complex numbers make it easier to analyze such waves.

Real-life example: "Wi-Fi signals are now part of our daily lives. The trigonometric form of complex numbers helps analyze signal strength, frequency, and phase."

3. 3D Graphics and Animation Software

In computer graphics and 3D animations, the trigonometric form is widely used for rotating objects, crucial for video games and visual effects.

Real-life example: "Popular 3D games like Fortnite and PUBG, and animated films, rely heavily on these mathematical ideas."

4. Marine Navigation and Radar Technology

In radar and sonar systems, complex numbers help determine direction and distance.

Real-life example: "Marine radar systems that help ships avoid collisions use complex numbers in trigonometric form to determine position and angle."

Effective Teaching Methods for Complex Numbers

To help students understand complex numbers, the following methods are particularly effective. These strategies encourage independent thinking and deepen knowledge.

1. Problem-Based Learning

Give students a real-life problem, such as: "An electrical engineer needs to calculate the phase difference between voltage and current in an AC circuit. How can this be done?"

Students then explore how trigonometric forms assist in solving this.

2. Observation and Analysis Method

Students observe how a technical device functions using complex numbers (e.g., current diagrams or sinusoidal waves) and analyze the formulas behind them.

3. Analogy Method

Explain complex numbers using familiar comparisons: "Think of complex numbers as points in a plane. The trigonometric form gives the angle — like a direction on a compass."

4. Project-Based Learning

Assign a mini-project, such as: "Model signal frequency, phase, and strength of a smartphone using complex numbers."

5. Interactive Visualization Method

Use tools like GeoGebra or Desmos to visualize how complex numbers rotate. Students can observe real-time changes in radius and angle.

Fields of Application

The trigonometric form of complex numbers is widely used in:

- Electrical engineering
- Signal and wave theory
- Mechanics and aerodynamics
- Mathematical physics
- Computer graphics

Conclusion. The trigonometric form of complex numbers is a powerful tool that integrates theoretical and practical aspects of mathematics. It simplifies complex operations into basic trigonometric expressions. De Moivre's formula, in particular, facilitates many technical and scientific calculations.

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