SHTURM-LIUVILL TENGLAMASINING XOSSALARI

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KALIT SO'ZLAR:

ANNOTATSIYA:

Bu yerda haqiqiy funksiya bo'lib, ixtiyoriy haqiqiy sonlar.

(1)+(2) Koshi masalasiga ekvivalent bo'ladigan integral tenglama tuzamiz. funksiya (1)+(2) masalaning biror yechimi bo'lsin. (1) tenglamani avvalo ushbu

Shturm-Liuvill tenglamasi uchun qo'yilgan Koshi masalasi

Quyidagi Koshi masalasini ko'rib chiqamiz:

$$-y'' + q(x)y = \lambda y, \quad 0 \le x \le \pi, \tag{1}$$

$$\begin{cases} y(0) = y_0, \\ y'(0) = y_1. \end{cases} \tag{2}$$

Bu yerda $q(x) \in C[0,\pi]$ haqiqiy funksiya bo'lib, y_0, y_1 ixtiyoriy haqiqiy sonlar.

(1)+(2) Koshi masalasiga ekvivalent bo'ladigan integral tenglama tuzamiz. $y(x)$ funksiya (1)+(2) masalaning biror yechimi bo'lsin. (1) tenglamani avvalo ushbu

$$y''(x) = f(x), \tag{3}$$

ko'rinishda yozib olamiz. Bu yerda

$$f(x) = [q(x) - \lambda]y(x). \tag{4}$$

So'ngra (3) tenglama uchun Koshi funksiyasini tuzamiz, ya'ni tarkibida t parametr qatnashgan ushbu

$$\begin{cases} y''=0, \\ y|_{x=t}=0, \\ y'|_{x=t}=1, \end{cases}$$

Koshi masalasini yechimini topamiz: $y=c_1x+c_2$

$$\begin{cases} c_1t+c_2=0, \\ c_1=1, \end{cases} \qquad \begin{cases} c_1=1, \\ c_2=-t, \end{cases}$$

$y=K(x,t)=x-t$

Shuning uchun (3) tenglamaning umumiy yechimi

$$y(x)=A_0+A_1x+\int_0^x (x-t)f(t)dt, \tag{5}$$

ko'rinishda bo'ladi. (1.44) boshlang'ich shartlardan

$$A_0=y_0, \quad A_1=y_1, \tag{6}$$

bo'lishi kelib chiqadi. (4) va (6) tengliklardan foydalanib, (5) ayniyatni ushbu

$$y(x)=y_0+y_1x+\int_0^x (x-t)\left[q(t)-\lambda\right]y(t)dt \tag{7}$$

ko'rinishda yozamiz. (7) tenglik izlangan integral tenglamadir. Bu tenglama Volterraning ikkinchi turdagi integral tenglamasidir.

Shunday qilib, (1)+(2) Koshi masalasining yechimi mavjud bo'lsa, u (7) integral tenglamani qanoatlantirar ekan. Aksincha, $y(x)$ funksiya (7) integral tenglamaning uzluksiz yechimi bo'lsa, u (1)+(2) Koshi masalasining ham yechimi bo'ladi. Haqiqatan ham, $y(x)$ uzluksiz ekanligidan (7) ayniyatning o'ng tomoni differensiallanuvchi bo'lishi kelib chiqadi, bundan esa chap tomon ham hosilaga ega bo'lishi ko'rinadi. Undan hosila olsak, ushbu

$$y'(x)=y_1+\int_0^x \left[q(t)-\lambda\right]y(t)dt \tag{8}$$

tenglik hosil bo'ladi. (8) ayniyatdan yana hosila olsak,

$$y''=\left[q(x)-\lambda\right]y(x),$$

ya'ni

$$-y''+q(x)y=\lambda y,$$

tenglik kelib chiqadi.(7) va (8) tengliklarda $x=0$ desak, (2) boshlang'ich shartlarni olamiz.

Teorema 1. Agar $q(x)\in C[0,\pi]$, y_0, $y_1\in R^1$ bo'lsa, u holda (1)+(2) Koshi masalasining $[0,\pi]$ kesmada aniqlangan $\varphi(x,\lambda)$ yechimi mavjud va yagona bo'lib, u x

o’zgaruvchining har bir tayinlangan qiymatida λ bo’yicha $\frac{1}{2}$ tartibdagi butun funksiyadir, ya’ni tayinlangan x da $\varphi(x,\lambda)$ funksiya kompleks tekislikning ixtiyoriy chegaralangan soxasida kompleks ma’noda differensiallanuvchidir.

Isbot.*(Mavjudliyi).* Koshi masalasi yechimining mavjudligini isbotlash uchun (7) integral tenglamaning uzluksiz yechimi mavjud ekanligini ko'rsatish yetarli. Buning uchun quyidagi funksiyalar ketma-ketligini tuzib olamiz:

$$\varphi_0(x,\lambda) = y_0 + y_1 x,$$
$$\varphi_n(x,\lambda) = y_0 + y_1 x + \int_0^x (x-t)\left[q(t)-\lambda\right]\varphi_{n-1}(t,\lambda)\,dt, \quad n \in N \tag{9}$$

Bu funksiyalar ketma-ketligi $x\in[0,\pi]$, $\lambda\in\mathbb{C}$ qiymatlarda aniqlangan. $R>0$ ixtiyoriy son bo’lsin. $\varphi_n(x,\lambda)$ ketma-ketlik $x\in[0,\pi]$, $|\lambda|\le R$ bo’lganda tekis yaqinlashishini ko’rsatamiz. Shu maqsadda ushbu

$$\varphi_0 + \sum_{n=1}^{\infty}[\varphi_n - \varphi_{n-1}], \tag{10}$$

funksional qatorni tuzib olamiz. Bu qatorning xususiy yig’indisi $\varphi_n(x,\lambda)$ funksiyaga teng bo’lishi ravshan. Quyidagi

$$M = \max_{[0,\pi]}|q(x)|, \ \ K = \max_{[0,\pi]}|\varphi_0(x,\lambda)|,$$

belgilashni kiritib olamiz. U holda ushbu

$$|\varphi_1(x,\lambda) - \varphi_0(x,\lambda)| = \left|\int_0^x (x-t)\left[q(t)-\lambda\right]\varphi_0(t,\lambda)\,dt\right| \le$$
$$\le \int_0^x \pi(M+R)K\,dt = \pi(M+R)Kx,$$

$$|\varphi_2(x,\lambda) - \varphi_1(x,\lambda)| = \left|\int_0^x (x-t)\left[q(t)-\lambda\right]\cdot\left[\varphi_1(t,\lambda)-\varphi_0(t,\lambda)\right]dt\right| \le$$
$$\le \left|\int_0^x (x-t)(M+R)\cdot\pi(M+R)K\cdot t\,dt\right| \le \pi^2(M+R)^2 K\frac{x^2}{2},$$

tengsizliklar o'rinli bodadi. Umuman $|\lambda|\le R,\ x\in[0,\pi]$ bo’lganida quyidagi

$$|\varphi_n(x,\lambda) - \varphi_{n-1}(x,\lambda)| \le \frac{K\left[x\pi(M+R)\right]^n}{n!}, \tag{11}$$

baholash o'rinli bo’ladi. Bu tengsizlik induksiya usulida osongina isbot qilinadi. (11) baholashga asosan ushbu

$$K+\sum_{n=1}^{\infty}K\frac{\left[\pi^{2}\left(M+R\right)\right]^{n}}{n!}<\infty,$$

Sonli qator (10) funksional qator uchun majoranta qator bo'ladi. Demak, $|\lambda|\le R,\ x\in[0,\pi]$ to'plamda (10) qator Veyershtrass alomatiga asosan tekis yaqinlashuvchi bo'ladi. Uning yig'indisini $\varphi(x,\lambda)$ orqali belgilaymiz. $\varphi_n(x,\lambda)$ funksiyalarning uzluksizligidan $\varphi(x,\lambda)$ funksiyaning uzluksizligi kelib chiqadi.

Agar (9) tenglikda $n\to\infty$ da limitga o'tsak;

$$\varphi(x,\lambda)=y_0+y_1x+\int_0^x(x-t)\left[q(t)-\lambda\right]\varphi(t,\lambda)\,dt,$$

ayniyat hosil bo'ladi. Demak, $\varphi(x,\lambda)$ funksiya (7) integral tenglamaning uzluksiz yechimi bo'lar ekan.

(*Yagonaligi*). Endi (7) integral tenglamaning yechimi yagona bo'lishini isbotlaymiz. Buning uchun ikkita $\varphi(x,\lambda)\neq\psi(x,\lambda)$ yechim mavjud deb faraz qilamiz. Bu yechimlarni integral tenglamaga qo'yib, hosil bo'lgan ayniyatlarni bir-biridan ayiramiz:

$$\varphi(x,\lambda)-\psi(x,\lambda)=\int_0^x(x-t)\left[q(t)-\lambda\right]\left[\varphi(t,\lambda)-\psi(t,\lambda)\right]dt.$$

Bu tenglikka asosan

$$\left|\varphi(x,\lambda)-\psi(x,\lambda)\right|\le\pi(M+R)\int_0^x\left|\varphi(t,\lambda)-\psi(t,\lambda)\right|dt, \qquad (12)$$

bo'ladi. Agar

$$z(x)=\int_0^x\left|\varphi(t,\lambda)-\psi(t,\lambda)\right|dt,$$

belgilash kiritsak, (12) tengsizlik ushbu

$$z'(x)-\pi(M+R)z(x)\le 0 \qquad (13)$$

ko'rinishni oladi. (13) tengsizlikni $e^{-\pi(M+R)x}$ funksiyaga ko'paytiramiz va chap tomonni ko'paytmaning hosilasi ko'rinishida yozamiz:

$$\left(z(x)e^{-\pi(M+R)x}\right)'\le 0 \qquad (14)$$

(14) tengsizlikda $x=t$ desak, va hosil bo'ladigan tengsizlikni $[0,\ x]$ oraliqda integrallasak,

$$z(x)\le 0$$

kelib chiqadi. (12) baholashdan

$$\left|\varphi(x,\lambda)-\psi(x,\lambda)\right|\le 0,$$

ya’ni $\varphi(x,\lambda)\equiv\psi(x,\lambda)$ kelib chiqadi. Bu esa farazimizga ziddir.

(*Butunligi*). $\varphi(x,\lambda)$ yechimning λ ga nisbatan butun funksiya bo’lishini isbotlaymiz. $\varphi_n(x,\lambda)$ funksiyalarning har biri $|\lambda|<R$ sohada golomorf bo’lishi Ravshan. Veyershtrassning kompleks analizdagi teoremasiga ko’ra $\varphi(x,\lambda)$ funksiya $|\lambda|<R$ sohada golomorf bo’ladi. $R>0$ son ixtiyoriy bo’lganligi uchun $\varphi(x,\lambda)$ butun funksiya bo’ladi. $\varphi(x,\lambda)$ ning $\frac{1}{2}$ tartibdagi butun funksiya bo’lishini keyinchalik yechimning asimptotikasidan foydalanib ko’rsatamiz.

Shturm-Liuvill tenglamasi yechimining asimptotikasi

Quyidagi Koshi masalasini ko’rib chiqamiz

$$-y''+q(x)y=\lambda y,\quad 0\le x\le\pi, \tag{15}$$

$$\begin{cases} y(0)=y_0, \\ y'(0)=y_1. \end{cases} \tag{16}$$

Bu yerda $q(x)\in C[0,\pi]$ haqiqiy funksiya bo’lib, y_0, y_1 haqiqiy sonlar.

(15)+(16) Koshi masalasining $y(x,\lambda)$ yechimi mavjud, yagona va λ ga nisbatan butun funksiya bo’lishini isbot qilgan edik.

Endi, $y(x,\lambda)$ yechimning $|\lambda|\to\infty$ bo’lganda asimptotikasini o’rganish maqsadida (15)-(16) Koshi masalasiga ekvivalent bo’lgan integral tenglama tuzamiz. Buning uchun avvalo (15) tenglamani ushbu

$$y''+\lambda y=f(x), \tag{17}$$

ko‘rinishda yozib olamiz. Bu yerda

$$f(x)=q(x)y(x,\lambda) \tag{18}$$

So’ngra (17) tenglama uchun Koshi funksiyasini tuzamiz, ya’ni tarkibida t parametr qatnashgan ushbu

$$\begin{cases} y''+\lambda y=0, \\ y\big|_{x=t}=0, \\ y'\big|_{x=t}=1, \end{cases}$$

Koshi masalasining yechimini topamiz:

$$y(x)=c_1\cos\sqrt{\lambda}x+c_2\frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}},$$

$$y'(x)=-c_1\sqrt{\lambda}\sin\sqrt{\lambda}x+c_2\cos\sqrt{\lambda}x,$$

$$\begin{cases} c_1 \cos\sqrt{\lambda}t + c_2 \dfrac{\sin\sqrt{\lambda}t}{\sqrt{\lambda}} = 0, \\ -c_1\sqrt{\lambda}\sin\sqrt{\lambda}t + c_2\cos\sqrt{\lambda}t = 1, \end{cases}$$

$$\Delta = \begin{vmatrix} \cos\sqrt{\lambda}t & \dfrac{\sin\sqrt{\lambda}t}{\sqrt{\lambda}} \\ -\sqrt{\lambda}\sin\sqrt{\lambda}t & \cos\sqrt{\lambda}t \end{vmatrix} = 1,$$

$$\Delta_1 = \begin{vmatrix} 0 & \dfrac{\sin\sqrt{\lambda}t}{\sqrt{\lambda}} \\ 1 & \cos\sqrt{\lambda}t \end{vmatrix} = -\frac{\sin\sqrt{\lambda}t}{\sqrt{\lambda}}$$

$$\Delta_2 = \begin{vmatrix} \cos\sqrt{\lambda}t & 0 \\ -\sqrt{\lambda}\sin\sqrt{\lambda}t & 1 \end{vmatrix} = \cos\sqrt{\lambda}t,$$

$$c_1 = -\frac{\sin\sqrt{\lambda}t}{\sqrt{\lambda}}, \quad c_2 = \cos\sqrt{\lambda}t,$$

$$K(x,t) = -\frac{\sin\sqrt{\lambda}t}{\sqrt{\lambda}}\cos\sqrt{\lambda}x + \cos\sqrt{\lambda}t\frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}} =$$

$$= \frac{1}{\sqrt{\lambda}}\sin\sqrt{\lambda}(x-t)$$

Koshi funksiyasi yordamida (17) tenglamaning umumiy yechimi quyidagicha ifodalanadi

$$y(x,\lambda) = A_0\cos\sqrt{\lambda}x + A_1\frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}} +$$

$$+\frac{1}{\sqrt{\lambda}}\int_0^x f(t)\sin\sqrt{\lambda}(x-t)dt. \qquad (19)$$

(18) belgilashni va boshlang‘ich shartlarni inobatga olsak, (19) tenglik ushbu

$$y(x,\lambda) = y_0\cos\sqrt{\lambda}x + y_1\frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}} +$$

$$+\frac{1}{\sqrt{\lambda}}\int_0^x q(t)y(t,\lambda)\sin\sqrt{\lambda}(x-t)dt, \qquad (20)$$

ko‘rinishni oladi. Bu tenglik biz izlagan integral tenglamadir. Bu tenglama Volterraning ikkinchi turdagi integral tenglamasidir, unga Liuvill integral tenglamasi deb ham aytiladi.

$c(x,\lambda)$ va $s(x,\lambda)$ orqali (15) tenglamaning quyidagi

$$\begin{cases} c(0,\lambda)=1, \\ c'(0,\lambda)=0 \end{cases} \quad \text{va} \quad \begin{cases} s(0,\lambda)=0, \\ s'(0,\lambda)=1 \end{cases}$$

boshlang‘ich shartlarini qanoatlantiruvchi yechimlarini belgilaymiz. Bu yechimlar uchun (20) integral tenglama ushbu

$$c(x,\lambda)=\cos\sqrt{\lambda}x+\frac{1}{\sqrt{\lambda}}\int_0^x q(t)c(t,\lambda)\sin\sqrt{\lambda}(x-t)dt, \tag{21}$$

$$s(x,\lambda)=\frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}}+\frac{1}{\sqrt{\lambda}}\int_0^x q(t)s(t,\lambda)\sin\sqrt{\lambda}(x-t)dt, \tag{22}$$

ko‘rinishlarda bo‘ladi.

Lemma 1.1. *Agar* $x\in[0,\pi]$, $\sqrt{\lambda}=k=\sigma+i\tau$, $|k|>2\int_0^\pi |q(t)|dt$ *bo’lsa, u holda*

$$|c(x,\lambda)|<2e^{|\tau|x}, \tag{23}$$

$$|s(x,\lambda)|<2\frac{e^{|\tau|x}}{|k|}, \tag{24}$$

baholashlar o’rinli bo’ladi.

Isbot. Quyidagi

$$F(x,\lambda)=\frac{c(x,\lambda)}{e^{|\tau|x}},$$

belgilashni kiritib olamiz. U holda

$$c(x,\lambda)=e^{|\tau|x}F(x,\lambda)$$

bo’ladi. Buni (21) tenglamaga qo’yamiz:

$$e^{|\tau|x}F(x,\lambda)=\cos kx+\frac{1}{k}\int_0^x q(t)e^{|\tau|t}F(t,\lambda)\sin k(x-t)dt,$$

$$F(x,\lambda)=\frac{\cos kx}{e^{|\tau|x}}+\frac{1}{k}\int_0^x q(t)F(t,\lambda)\frac{\sin k(x-t)}{e^{|\tau|(x-t)}}dt \tag{25}$$

Quyidagi baholashlarni bajaramiz:

$$|\cos kx|=\left|\frac{e^{ikx}+e^{-ikx}}{2}\right|=\left|\frac{e^{i\sigma x-\tau x}+e^{-i\sigma x+\tau x}}{2}\right|\le\frac{1}{2}\left(e^{-\tau x}+e^{\tau x}\right)\le e^{|\tau|x},$$

$$|\sin kx|=\left|\frac{e^{ikx}-e^{-ikx}}{2i}\right|=\left|\frac{e^{i\sigma x-\tau x}-e^{-i\sigma x+\tau x}}{2}\right|\le\frac{1}{2}\left(e^{-\tau x}+e^{\tau x}\right)\le e^{|\tau|x}.$$

$M(\lambda)=\max\limits_{0\le x\le\pi}|F(x,\lambda)|$ bo’lsin. U holda yuqorida olingan baholashlarga ko’ra (25) tenglikdan ushbu

$$\left|F(x,\lambda)\right| \le 1+\frac{1}{|k|}\int_0^{\pi}\left|q(t)\right|M(\lambda)\,dt,$$

tengsizlik kelib chiqadi. Bu tengsizlikdan esa

$$M(\lambda)\le 1+M(\lambda)\cdot\frac{1}{|k|}\int_0^{\pi}\left|q(t)\right|dt,$$

tengsizlik kelib chiqadi. Bu tengsizlikdan esa

$$M(\lambda)\le 1+M(\lambda)\cdot\frac{1}{|k|}\int_0^{\pi}\left|q(t)\right|dt,$$

ya'ni

$$M(\lambda)\left(1-\frac{1}{|k|}\int_0^{\pi}\left|q(t)\right|dt\right)\le 1, \tag{26}$$

hosil bo'ladi. Lemma shartiga ko'ra

$$\frac{1}{|k|}\int_0^{\pi}\left|q(t)\right|dt<\frac{1}{2},$$

$$1-\frac{1}{|k|}\int_0^{\pi}\left|q(t)\right|dt>\frac{1}{2}, \tag{27}$$

bo'ladi. (26) va (27) tengsizliklardan ushbu $M(\lambda)<2$ baholash kelib chiqadi, ya'ni $\left|F(x,\lambda)\right|<2$ bo'ladi. Shunday qilib (23) baholash isbot qilindi. (24) baholash ham shu tarzda isbot qilinadi.

Lemma 1.2. *Agar* $x\in[0,\pi]$, $\sqrt{\lambda}=k=\sigma+i\tau$, $|k|>2\int_0^{\pi}\left|q(t)\right|dt$ *bo'lsa, u holda*

$$\left|c(x,\lambda)-\cos\sqrt{\lambda}x\right|\le\frac{2}{|k|}\int_0^{x}\left|q(t)\right|dt\cdot e^{|\tau|x}, \tag{28}$$

$$\left|s(x,\lambda)-\frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}}\right|\le\frac{2}{|k|^2}\int_0^{x}\left|q(t)\right|dt\cdot e^{|\tau|x}, \tag{29}$$

$$\left|c'(x,\lambda)+\sqrt{\lambda}\sin\sqrt{\lambda}x\right|\le 2\int_0^{x}\left|q(t)\right|dt\cdot e^{|\tau|x}, \tag{30}$$

$$\left|s'(x,\lambda)-\cos\sqrt{\lambda}x\right|\le\frac{2}{|k|}\int_0^{x}\left|q(t)\right|dt\cdot e^{|\tau|x}, \tag{31}$$

baholashlar o'rinli bo'ladi.

Isbot. (21) integral tenglama va (23) tengsizlikka ko'ra

$$\left|c(x,\lambda)-\cos\sqrt{\lambda}x\right|\le\frac{1}{|k|}\int_0^x|q(t)|\cdot|c(t,\lambda)|\cdot|\sin k(x-t)|dt\le$$

$$\le\frac{1}{|k|}\int_0^x|q(t)|\cdot 2e^{|\tau|t}\cdot e^{|\tau|(x-t)}dt\le\frac{2}{|k|}\int_0^x|q(t)|dt\cdot e^{|\tau|x},$$

bo’ladi. (22) integral tenglama va (24) baholashga ko’ra

$$\left|s(x,\lambda)-\frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}}\right|\le\frac{1}{|k|}\int_0^x|q(t)|\cdot|s(t,\lambda)|\cdot|\sin k(x-t)|dt\le$$

$$\le\frac{1}{|k|}\int_0^x|q(t)|\cdot 2\frac{e^{|\tau|t}}{|k|}\cdot e^{|\tau|(x-t)}dt\le\frac{2}{|k|^2}\int_0^x|q(t)|dt\cdot e^{|\tau|x},$$

bo’ladi. (28) va (29) baholashlar isbotlandi. (30) va (31) tengsizliklarni isbot qilish maqsadida, avvalo (21) va (22) tenglamalardan hosila olamiz:

$$c'(x,\lambda)=-\sqrt{\lambda}\sin\sqrt{\lambda}x+\int_0^x q(t)c(t,\lambda)\cos\sqrt{\lambda}(x-t)dt,$$

$$s'(x,\lambda)=\cos\sqrt{\lambda}x+\int_0^x q(t)s(t,\lambda)\cos\sqrt{\lambda}(x-t)dt.$$

Bu tengliklardan hamda (23) va (24) tengsizliklardan foydalanib, ushbu

$$\left|c'(x,\lambda)+\sqrt{\lambda}\sin\sqrt{\lambda}x\right|\le\int_0^x|q(t)|\cdot|c(t,\lambda)|\cdot|\cos k(x-t)|dt\le$$

$$\le\int_0^x|q(t)|\cdot 2e^{|\tau|t}\cdot e^{|\tau|(x-t)}dt\le 2\int_0^x|q(t)|dt\cdot e^{|\tau|x},$$

$$\left|s'(x,\lambda)-\cos\sqrt{\lambda}x\right|\le\int_0^x|q(t)|\cdot|s(t,\lambda)|\cdot|\cos k(x-t)|dt\le$$

$$\le\int_0^x|q(t)|\cdot 2\frac{e^{|\tau|t}}{|k|}\cdot e^{|\tau|(x-t)}dt\le\frac{2}{|k|}\int_0^x|q(t)|dt\cdot e^{|\tau|x},$$

baholashlarni olamiz.

Natija 1.5. Agar $x\in[0,\pi]$, $\sqrt{\lambda}=k=\sigma+i\tau$, $|k|>2\int_0^\pi|q(t)|dt$ bo’lsa, u holda quyidagi

$$c(x,\lambda)=\cos\sqrt{\lambda}x+\underline{\underline{O}}\left(\frac{e^{|\tau|x}}{k}\right),$$

$$s(x,\lambda)=\frac{\sin\sqrt{\lambda}x}{\sqrt{\lambda}}+\underline{\underline{O}}\left(\frac{e^{|\tau|x}}{k^2}\right),$$

$$c'(x,\lambda)=-\sqrt{\lambda}\sin\sqrt{\lambda}x+\underline{\underline{O}}\left(e^{|\tau|x}\right),$$

$$s'(x,\lambda)=\cos\sqrt{\lambda}x+\underline{\underline{O}}\left(\frac{e^{|\tau|x}}{k}\right),$$

asimptotik formulalar o'rinli bo'ladi.

Natija 1.6. Agar $\varphi(x,\lambda)$ orqali (15) tenglamaning ushbu

$$\varphi(0,\lambda)=1,\quad \varphi'(0,\lambda)=h$$

boshlang'ich shartlarni qanoatlantiruvchi yechimini belgilasak, u holda

$$\varphi(x,\lambda)=c(x,\lambda)+hs(x,\lambda)$$

bo'ladi. Bundan $x\in[0,\pi]$, $\sqrt{\lambda}=k=\sigma+i\tau$, $|k|>2\int_0^\pi|q(t)|dt$ bo'lganda ushbu

$$\varphi(x,\lambda)=\cos\sqrt{\lambda}x+\underline{\underline{O}}\left(\frac{e^{|\mathrm{Im}\sqrt{\lambda}|x}}{\sqrt{\lambda}}\right),$$

$$\varphi'(x,\lambda)=-\sqrt{\lambda}\sin\sqrt{\lambda}x+\underline{\underline{O}}\left(e^{|\mathrm{Im}\sqrt{\lambda}|x}\right),$$

asimptotik formulalar o'rinli bo'lishi kelib chiqadi.

Foydalanilgan adabiyotlar:

1. Carroll R. Transmutation and Operator Differential Equations.-North Holland, 1979.-245 p.
2. Carroll R. Transmutation, Scattering Theory and Special Functions. –North Holland, 1982.-457 p.
3. Carroll R. Transmutation Theory and Applications.-North Holland, 1986.-351p.
4. G. Freiling and V. Yurko.Inverse Sturm-Liouville problems and their applications. 13 (1997), 1247-1263.
5. Gilbert R., Begehr H. Transformations, Transmutations and Kernel Functions. Vol. 1–2.-Longman, Pitman, 1992.

6. Trimeche Kh. Transmutation Operators and Mean-Periodic Functions Associated with Differential Operators (Mathematical Reports, Vol 4, Part 1).-Harwood Academic Publishers, 1988.-282 p.

7. A.B. Hasanov Shturm-Liuvill chegaraviy masalalari nazariyasiga kirish. Toshkent: “Turon-Iqbol”, 2016.-584 b.

8. Фаге Д. К., Нагнибида Н. И. Проблема эквивалентности обыкновенных дифференциальных операторов. -Новосибирск: Наука, 1977.-280 с.

9. Gilbert R. Constructive Methods for Elliptic Equations. Springer Lecture Notes Math, 365, 1974.

10. Ситник С.М., Шишкина Э.Л. Метод операторов преобразования для дифференциальных уравнений с операторами Бесселя. 2019, ФИЗМАТЛИТ, 250 стр.