JOURNAL OF INTERNATIONAL SCIENTIFIC RESEARCH
Volume 3, Issue 9, February 2026 Online ISSN: 3030-3508
https://spaceknowladge.com

# HOSILA VA UNING TADBIQLARI

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**MAQOLA TARIXI:**

*Received:24.02.2026*
*Revised: 25.02.2026*
*Accepted:26.02.2026*

**KALIT SO'ZLAR:**

*hosila, limit, differensial, ekstremum, infleksiya nuqtasi, optimallashtirish, marjinal tahlil, tezlik, tezlanish, differensial tenglama, Teylor formulasi.*

**ANNOTATSIYA:**

*Mazkur ilmiy ishda hosila tushunchasining nazariy asoslari, limit orqali aniqlanishi, geometrik va fizik mazmuni, differensial tushunchasi, yuqori tartibli hosilalar, Teylor formulasi elementlari hamda hosilaning matematik, iqtisodiy, fizik, texnik va biologik jarayonlardagi qo'llanilishi keng va chuqur tahlil qilinadi. Funksiyalarning o'sish-kamayish oraliqlari, ekstremumlari, qavariqlik xossalari, infleksiya nuqtalari, optimallashtirish masalalari va differensial tenglamalar orqali modellashtirish jarayonlari batafsil misollar asosida yoritiladi. Ish matematik analizning fundamental tushunchalaridan biri bo'lgan hosilaning nazariy va amaliy ahamiyatini kompleks ravishda ochib beradi.*

Hosila matematik analiz fanining eng muhim va markaziy tushunchalaridan biridir. Differensial hisob XVII asrda mustaqil ravishda Isaac Newton va Gottfried Wilhelm Leibniz tomonidan yaratilgan bo'lib, u ilm-fan taraqqiyotida tub burilish yasagan. Newton hosilani harakat nazariyasida qo'llagan bo'lsa, Leibniz uni umumiy matematik apparat sifatida shakllantirgan va bugungi kunda qo'llanilayotgan dydx\frac{dy}{dx}dxdy belgilar tizimini joriy etgan.

Hosilaning asosiy ta'rifi limit tushunchasiga asoslanadi. Agar y=f(x)y=f(x)y=f(x) funksiya berilgan bo'lsa, uning hosilasi quyidagicha aniqlanadi:

f′(x)=lim⁡Δx→0f(x+Δx)−f(x)Δxf'(x)=\lim_{\Delta x \to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}f′(x)=Δx→0limΔxf(x+Δx)−f(x)

Bu ifoda funksiyaning o‘rtacha o‘zgarish tezligining limitdagi qiymatini bildiradi. Masalan, f(x)=x2f(x)=x^2f(x)=x2 bo‘lsa:

f(x+Δx)−f(x)Δx=(x+Δx)2−x2Δx=2xΔx+(Δx)2Δx=2x+Δx\frac{f(x+\Delta x)-f(x)}{\Delta x} =\frac{(x+\Delta x)^2-x^2}{\Delta x} =\frac{2x\Delta x+(\Delta x)^2}{\Delta x} =2x+\Delta xΔxf(x+Δx)−f(x)=Δx(x+Δx)2−x2=Δx2xΔx+(Δx)2=2x+Δx lim⁡Δx→0(2x+Δx)=2x\lim_{\Delta x \to 0}(2x+\Delta x)=2xΔx→0lim(2x+Δx)=2x

Demak,

(x2)′=2x(x^2)'=2x(x2)′=2x

Xuddi shunday:

(x3)′=3x2,(x4)′=4x3(x^3)'=3x^2, \quad (x^4)'=4x^3(x3)′=3x2,(x4)′=4x3

Umuman:

(xn)′=nxn−1(x^n)'=nx^{n-1}(xn)′=nxn−1

Hosilaning geometrik mazmuni egri chiziqqa o‘tkazilgan urinmaning qiyalik koeffitsiyenti bilan ifodalanadi. Agar f′(x0)=kf'(x_0)=kf′(x0)=k bo‘lsa, urinma tenglamasi:

y−y0=k(x−x0)y-y_0=k(x-x_0)y−y0=k(x−x0)

Misol: f(x)=x2f(x)=x^2f(x)=x2, x0=1x_0=1x0=1

f′(x)=2x⇒f′(1)=2f'(x)=2x \Rightarrow f'(1)=2f′(x)=2x⇒f′(1)=2 y−1=2(x−1)y-1=2(x-1)y−1=2(x−1) y=2x−1y=2x-1y=2x−1

Hosilaning fizik mazmuni mexanikada yaqqol ko‘rinadi. Agar:

s(t)=5t2+3ts(t)=5t^2+3ts(t)=5t2+3t

bo‘lsa,

v(t)=s′(t)=10t+3v(t)=s'(t)=10t+3v(t)=s′(t)=10t+3 a(t)=v′(t)=10a(t)=v'(t)=10a(t)=v′(t)=10

Demak, tezlanish doimiy.

Hosila olishning asosiy qoidalari:

(C)′=0(C)'=0(C)′=0 (f+g)′=f′+g′(f+g)'=f'+g'(f+g)′=f′+g′ (fg)′=f′g+fg′(fg)'=f'g+fg'(fg)′=f′g+fg′ (fg)′=f′g−fg′g2\left(\frac{f}{g}\right)'=\frac{f'g-fg'}{g^2}(gf)′=g2f′g−fg′ (f(g(x)))′=f′(g(x))⋅g′(x)(f(g(x)))'=f'(g(x))\cdot g'(x)(f(g(x)))′=f′(g(x))⋅g′(x)

Misol:

y=(3x2+1)(x3−2)y=(3x^2+1)(x^3-2)y=(3x2+1)(x3−2)

y′=6x(x3−2)+(3x2+1)(3x2)y'=6x(x^3-2)+(3x^2+1)(3x^2)y′=6x(x3−2)+(3x2+1)(3x2)

Trigonometrik funksiyalar:

(sin⁡x)′=cos⁡x(\sin x)'=\cos x(sinx)′=cosx (cos⁡x)′=−sin⁡x(\cos x)'=-\sin x(cosx)′=−sinx (tan⁡x)′=1cos⁡2x(\tan x)'=\frac{1}{\cos^2 x}(tanx)′=cos2x1

Misol:

y=x2sin⁡xy=x^2\sin xy=x2sinx y′=2xsin⁡x+x2cos⁡xy'=2x\sin x+x^2\cos xy′=2xsinx+x2cosx

Ko‘rsatkichli va logarifmik funksiyalar:

(ex)′=ex(e^x)'=e^x(ex)′=ex (ax)′=axln⁡a(a^x)'=a^x\ln a(ax)′=axlna (ln⁡x)′=1x(\ln x)'=\frac{1}{x}(lnx)′=x1

Murakkab funksiya misoli:

y=sin⁡(x2)y=\sin(x^2)y=sin(x2) y′=cos⁡(x2)·2xy'=\cos(x^2)\cdot2xy′=cos(x2)·2x

Yuqori tartibli hosilalar:

f′(x)=ddx(f′(x))f''(x)=\frac{d}{dx}(f'(x))f′′(x)=dxd(f′(x))

Misol:

f(x)=x3f(x)=x^3f(x)=x3 f′(x)=3x2f'(x)=3x^2f′(x)=3x2 f′′(x)=6xf''(x)=6xf′′(x)=6x f′′′(x)=6f'''(x)=6f′′′(x)=6

Ikkinchi hosila yordamida qavariqlik aniqlanadi.

Ekstremum sharti:

f′(x)=0f'(x)=0f′(x)=0

Misol:

f(x)=x3−3xf(x)=x^3-3xf(x)=x3−3x f′(x)=3x2−3=3(x2−1)f'(x)=3x^2-3=3(x^2-1)f′(x)=3x2−3=3(x2−1) x=±1x=\pm1x=±1 f′′(x)=6xf''(x)=6xf′′(x)=6x f′′(−1)=−6<0⇒maksimumf''(-1)=-6<0 \Rightarrow maksimumf′′(−1)=−6<0⇒maksimum f′′(1)=6>0⇒minimumf''(1)=6>0 \Rightarrow minimumf′′(1)=6>0⇒minimum

Optimallashtirish misoli. Perimetri 20 bo‘lgan to‘rtburchak yuzasi:

2x+2y=202x+2y=202x+2y=20 y=10−xy=10-xy=10−x S=x(10−x)=10x−x2S=x(10-x)=10x-x^2S=x(10−x)=10x−x2 S′=10−2x=0S'=10-2x=0S′=10−2x=0 x=5x=5x=5

Demak maksimal yuzaga ega shakl kvadrat.

Iqtisodiy misol. Xarajat funksiyasi:

C(x)=x3−6x2+15xC(x)=x^3-6x^2+15xC(x)=x3−6x2+15x MC=C′(x)=3x2−12x+15MC=C'(x)=3x^2-12x+15MC=C′(x)=3x2−12x+15

Foyda:

P(x)=R(x)−C(x)P(x)=R(x)-C(x)P(x)=R(x)−C(x) P′(x)=0P'(x)=0P′(x)=0

sharti maksimal foyda beradi.

Biologik model:

dPdt=kP\frac{dP}{dt}=kPdtdP=kP

Yechimi:

P(t)=P0ektP(t)=P_0e^{kt}P(t)=P0ekt

Radioaktiv parchalanish:

dNdt=−λN\frac{dN}{dt}=-\lambda NdtdN=−λN N=N0e−λtN=N_0e^{-\lambda t}N=N0e−λt

Teylor formulasi elementlari:

f(x)=f(a)+f′(a)(x−a)+f′′(a)2!(x−a)2+…f(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\dotsf(x)=f(a)+f′(a)(x−a)+2!f′′(a)(x−a)2+…

Masalan exe^xex uchun:

ex=1+x+x22!+x33!+…e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dotsex=1+x+2!x2+3!x3+…

Hosila yordamida funksiyani to‘liq tekshirish algoritmi:

1. Aniqlanish sohasi
2. Birinchi hosila
3. Kritik nuqtalar
4. Ikkinchi hosila
5. Ekstremum va infleksiya
6. Chegaraviy xatti-harakat
7. Grafik chizish

Ko‘plab fizik qonunlar differensial tenglama ko‘rinishida ifodalanadi. Masalan, Nyutonning ikkinchi qonuni:

F=ma=md2xdt2F=ma=m\frac{d^2x}{dt^2}F=ma=mdt2d2x

Issiqlik o‘tkazuvchanlik tenglamalari, to‘lqin tenglamalari va elektr zanjir tenglamalari ham hosila asosida quriladi.

Xulosa qilib aytganda, hosila matematik analizning fundamental tushunchasi bo‘lib, u funksiyalarning lokal xatti-harakatini o‘rganish, optimallashtirish masalalarini hal qilish, fizik va iqtisodiy jarayonlarni modellashtirish hamda differensial tenglamalar tuzishda asosiy vosita hisoblanadi. Hosila tushunchasi zamonaviy fan-texnikaning matematik tayanchidir va uning qo‘llanilish sohasi nihoyatda kengdir.

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