MAKSIMAL HIPONILPOTENT IDEALGA EGA YETTI O'LCHAMLI YECHILUVCHAN 3-LI ALGEBRALARI TASNIFI

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MAQOLA MA'LUMOTI

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KALIT SO'ZLAR:
n-Li algebralar, nilpotent n-Li algebralar, hiponilpotent algebralar, yechiluvchan n-algebralar, differensial.

ANNOTATSIYA:

Dastlab 1985-yilda Filippov[9] n -Li algebrasi tushunchasini kiritdi va (n+1) -o'lchamli n -Li algebralarini tasnifladi. 2009-yilda R.Bai va boshqalar[3] n -Li algebralarida giponilpotent ideal tushunchasini kiritdilar va berilgan maksimal m -o'lchamli filiform giponilpotent ideallar bilan yechiluvchan 3-Li algebralarining tavsifini oldilar. Ushbu maqolada maksimal filiform hiponilpotent idealga ega bo'lgan maksimal yechiluvchan 3-Li algebralari tasnif qilingan.

KIRISH.

n-Li algebralarining dinamik sistemalar, geometriya va fizikaning turli sohalardagi tadbiqlari tufayli bu algebralarni o'rganish muhim ahamiyat kasb etmoqda. Maksimal hiponilpotent idealga ega bo'lgan chekli o'lchamli yechiluvchan 3-Li algebralarini tasniflash metodini chekli o'lchamli yechiluvchan 3-Li algebralari uchun tadbiq etish maqsadli ilmiy tadqiqotlardan biridir.

2009-yilda R.Bai va boshqalar[9] n-Li algebralarida hiponilpotent ideal tushunchasini kiritdilar va berilgan maksimal m-o'lchamli filiform hiponilpotent ideallar bilan yechiluvchan 3-Li algebralarining tavsifini oldilar. Shuningdek, ular bunday yechiluvchan 3-Li algebralarining o'lchami eng ko'p $m+2$ ekanligini isbotladilar. Bundan tashqari, m-o'lchamli filiform 3-Li algebrasi yechiluvchan nilpotent bo'lmagan 3-Li algebrasining nilradikali bo'lishi mumkin emasligi isbotladilar.

Dastlabki ma’lumotlar

1-ta’rif.[1] F maydon ustida aniqlangan L vektor fazoda shunday n-ar poli-chiziqli $[-,-,\dots,-]$amal mavjud bo‘lib, quyidagi ayniyatlarni qanoatlantirsa:

$$[x_1,\dots,x_n]=(-1)^{sign(\sigma)}[x_{\sigma(1)},\dots,x_{\sigma(n)}],$$

va

$$[[x_1,\dots,x_n],y_2,\dots,y_n]=\sum_{i=1}^{n}[x_1,\dots,[x_i,y_2,\dots,y_n],\dots,x_n]$$

u holda L algebraga n-*Li algebrasi* deyiladi, bu yerda $\forall x_1,\dots,x_n,y_2,\dots,y_n\in L$ va ixtiyoriy $\sigma\in S_n$ va $sign(\sigma)-\sigma$ o’rinlashtirishning juft yoki toqligi.

2-ta’rif.[1] Aytaylik, L n-Li algebrasining $D:L\to L$ chiziqli akslantirishi berilgan bo‘lsin. Agar ixtiyoriy $x_1,x_2,\dots,x_n\in L$ elementlari uchun, quyidagi tenglik o‘rinli bo‘lsa,

$$D([x_1,x_2,\dots,x_n])=\sum_{i=1}^{n}[x_1,\dots,D(x_i),\dots,x_n],$$

u holda D chiziqli akslantirishga L n-Li algebrasining *differensiallashi* deyiladi. L n-Li algebrasining barcha differensiallashlari to‘plami $Der(L)$ kabi belgilanadi va u $gl(L)$ Li algebrasining qism algebrasi bo‘ladi.

Agar $ad(x_2,x_3,\dots,x_n):L\to L$ akslantirish quyidagi tenglikni qanoatlantirsa,

$$ad(x_2,x_3,\dots,x_n)(y)=[y,x_2,x_3,\dots,x_n]\ \ \forall\ y\in L,$$

u holda ushbu akslantirishga *o’ng ko’paytma* deyiladi.

Bu ko’paytma L n-Li algebrasi uchun differensiallash bo’ladi. O‘ng ko’paytma operatorlarining barcha chiziqli kombinatsiyalari $Der(L)$ Li algebrasining ideali bo’ladi va $Ad(L)$ fazoning elementlari *ichki differensiallashlar* deyiladi.

3-ta’rif.[12] L n-Li algebrasi va B uning qism fazosi bo’lsin. Agar $[B,B,\dots,B]\subseteq B$ munosabati o’rinli bo’lsa, u holda B n-Li algebrasining *qism algebrasi* deyiladi.

4-ta’rif.[12] L n-Li algebrasining I qism fazosi uchun $[I,L,\dots,L]\subseteq I$ munosabat o’rinli bo’lsa, I n-Li algebrasining *ideali* deyiladi.

L n-Li algebrasining ixtiyoriy I ideali uchun mos ravishda quyi markaziy qator va hosilaviy qatorlarni quyidagicha aniqlaymiz:

$$I^1=I,I^{k+1}=[I^k,I,L,\dots,L],k\ge 1,$$

$$I^{(1)}=I,I^{(s+1)}=[I^{(s)},I^{(s)},L,\dots,L],s\ge 1,$$

5-ta'rif.[1] L n-Li algebrasi bo'lsin. Agar $\exists r \in N$ soni uchun $I^{(r)} = 0$ munosabat o'rinli bo'lsa, I *yechiluvchan ideal* deyiladi, xususan agar $\exists r \in N$ soni uchun $L^{(r)} = 0$ tenglik o'rinli bo'lsa, u holda L n-Li algebrasi *yechiluvchan* n*-Li algebrasi* deyiladi.

6-ta'rif.[1] Agar $\exists r \in N$ soni uchun $I^r = 0$ bo‘lsa, u holda I nilpotent ideal deyiladi. Agar $\exists r \in N$ soni uchun $L^r = 0$ tenglik o‘rinli bo‘lsa, u holda L *nilpotent* n*-Li algebra* deyiladi.

7-ta'rif.[12] Agar m o‘lchamli L n-Li algebrasi uchun $dimL^i = m - n + 2 - i,\ 2 \leq i \leq m - n + 2$ tenglik o‘rinli bo‘lsa, ushbu algebra *filiform algebrasi* deyiladi.

Aytaylik, L n-Li algebrasi va I uning ideali bo‘lsin.

- I -nilpotent ideal $\Leftrightarrow \exists k \in N : [I^{k-1}, I, L, ..., L] = 0$
- I -nilpotent qism algebra $\Leftrightarrow \exists s \in N : [I^{s-1}, I, I, ..., I] = 0$

8-ta'rif.[1] Agar I nilpotent qism algebrasi bo'lib, lekin nilpotent ideal bo'lmasa, u holda I idealga L n-Li algebrasining *hiponilpotent ideali* deyiladi.

Agar I hiponilpotent idealni o‘z ichiga oluvchi I dan boshqa hiponilpotent ideal mavjud bo'lmasa, I *maksimal hiponilpotent ideal* deyiladi.

7 o‘lchamli 3-Li algebralarining differensiallashlar fazosi

7 o‘lchamli 3-Li algebraning differensiallashlar fazosini ko‘rib chiqaylik:

$$\mathcal{N}': \{[e_1, e_2, e_i] = e_{i+2}, \quad 3 \leq i \leq 5$$

$$Der(\mathcal{N}'): \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} & a_{1,5} & a_{1,6} & a_{1,7} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} & a_{2,5} & a_{2,6} & a_{2,7} \\ 0 & 0 & a_{3,3} & a_{3,4} & a_{3,5} & a_{3,6} & a_{3,7} \\ 0 & 0 & 0 & a_{4,4} & a_{4,5} & a_{4,6} & a_{4,7} \\ 0 & 0 & 0 & 0 & \delta + a_{3,3} & a_{3,4} & a_{3,5} \\ 0 & 0 & 0 & 0 & 0 & \delta + a_{4,4} & a_{4,5} \\ 0 & 0 & 0 & 0 & 0 & 0 & 2\delta + a_{3,3} \end{pmatrix},$$

bu yerda $\delta = a_{1,1} + a_{2,2}$.

$e_1' = e_1, e_2' = \alpha e_1 + e_2, e_i' = e_i, \quad 3 \leq i \leq 7$ bazis almashtirish bajarilsa, quyidagi matritsa hosil bo‘ladi:

$$Der(\mathcal{N}'):\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} & a_{1,5} & a_{1,6} & a_{1,7} \\ 0 & a_{2,2} & a_{2,3} & a_{2,4} & a_{2,5} & a_{2,6} & a_{2,7} \\ 0 & 0 & a_{3,3} & a_{3,4} & a_{3,5} & a_{3,6} & a_{3,7} \\ 0 & 0 & 0 & a_{4,4} & a_{4,5} & a_{4,6} & a_{4,7} \\ 0 & 0 & 0 & 0 & \delta + a_{3,3} & a_{3,4} & a_{3,5} \\ 0 & 0 & 0 & 0 & 0 & \delta + a_{4,4} & a_{4,5} \\ 0 & 0 & 0 & 0 & 0 & 0 & 2\delta + a_{3,3} \end{pmatrix},$$

bu yerda $\delta = a_{1,1} + a_{2,2}$.

$\mathcal{N}'$ algebraning ichki differensillashlar fazosi quyidagi matritsa ko‘rinishida bo‘ladi:

$$InDer(\mathcal{N}'):\begin{pmatrix} 0 & 0 & 0 & 0 & \gamma_1 & \gamma_2 & \gamma_3 \\ 0 & 0 & 0 & 0 & -\beta_1 & -\beta_2 & -\beta_3 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Maksimal hiponilpotent idealga ega o‘n o‘lchamli yechiluvchan 3-Li algebralari tasnifi

1-teorema.[12] R maksimal hiponilpotent idealga ega 10-o‘lchamli yechiluvchan 3-Li algebrasi bo’lsin. U holda R da $\{x, y, z, e_1, e_2, \dots, e_7\}$ bazis mavjud va $\mathcal{R}$ ning ko‘paytmalari quyidagiga teng:

$$\mathcal{R}:\begin{cases} [e_1,e_2,e_3]=e_5, & [x_1,e_1,e_7]=2e_7, \\ [e_1,e_2,e_4]=e_6, & [x_2,e_1,e_3]=e_3, \\ [e_1,e_2,e_5]=e_7, & [x_2,e_1,e_5]=e_5, \\ [x_1,e_1,e_2]=e_2, & [x_2,e_1,e_7]=e_7, \\ [x_1,e_1,e_5]=e_5, & [x_3,e_1,e_4]=e_4, \\ [x_1,e_1,e_6]=e_6, & [x_3,e_1,e_6]=e_6. \end{cases}$$

va bazis elementlarining qolgan ko‘paytmalari nolga teng.

Isbot. D_1, D_2, D_3, D_4-chiziqli erkli differensiallashlar

$$ad(x_1,e_1)(e_i) = D_2(e_i), ad(x_2,e_1)(e_i) = D_3(e_i), ad(x_3,e_1)(e_i) = D_4(e_i)$$

Quyidagi ko‘paytmalarni yozib olamiz:

$$[x_1,e_1,e_2] = e_2 + \alpha_{23}e_3 + \alpha_{24}e_4 + \alpha_{25}e_5 + \alpha_{26}e_6 + \alpha_{27}e_7,$$
$$[x_1,e_1,e_3] = \alpha_{34}e_4 + \alpha_{35}e_5 + \alpha_{36}e_6 + \alpha_{37}e_7,$$
$$[x_1,e_1,e_4] = \alpha_{45}e_5 + \alpha_{46}e_6 + \alpha_{47}e_7,$$

$[x_1,e_1,e_5]=e_5+\alpha_{34}e_6+\alpha_{35}e_7,$
$[x_1,e_1,e_6]=e_6+\alpha_{45}e_7,$
$[x_1,e_1,e_7]=2e_7,$
$[x_2,e_1,e_2]=\alpha_{23}e_3+\alpha_{24}e_4+\alpha_{25}e_5+\alpha_{26}e_6+\alpha_{27}e_7,$
$[x_2,e_1,e_3]=e_3+\alpha_{34}e_4+\alpha_{35}e_5+\alpha_{36}e_6+\alpha_{37}e_7,$
$[x_2,e_1,e_4]=\alpha_{45}e_5+\alpha_{46}e_6+\alpha_{47}e_7,$
$[x_2,e_1,e_5]=e_5+\alpha_{34}e_6+\alpha_{35}e_7,$
$[x_2,e_1,e_6]=\alpha_{45}e_7,$
$[x_2,e_1,e_7]=e_7,$
$[x_3,e_1,e_2]=\alpha_{23}e_3+\alpha_{24}e_4+\alpha_{25}e_5+\alpha_{26}e_6+\alpha_{27}e_7,$
$[x_3,e_1,e_3]=\alpha_{34}e_4+\alpha_{35}e_5+\alpha_{36}e_6+\alpha_{37}e_7,$
$[x_3,e_1,e_4]=e_4+\alpha_{45}e_5+\alpha_{46}e_6+\alpha_{47}e_7,$
$[x_3,e_1,e_5]=\alpha_{34}e_6+\alpha_{35}e_7,$
$[x_3,e_1,e_6]=e_6+\alpha_{45}e_7.$

3-Li algebralari uchun Umumlashgan Yakobi ayniyati yordamida ba’zi koeffitsiyentlarni nolga tengligini olamiz.

$[[x_1,e_1,e_2],e_1,x_2]$ ushbu ko‘paytma uchun quyidagi tenglik o‘rinli bo‘ladi:

$$[[x_1,e_1,e_2],x_2,e_1]=[[x_1,x_2,e_1],e_1,e_2]+[x_1,[e_1,x_2,e_1],e_2]+[x_1,e_1,[e_2,x_2,e_1]]$$

Tenglikning chap tomoni quyidagi tenglikni qanoatlantiradi:

$$[[x_1,e_1,e_2],x_2,e_1]=[e_2+\alpha_{23}e_3+\alpha_{24}e_4+\alpha_{25}e_5+\alpha_{26}e_6+\alpha_{27}e_7,x_2,e_1]=\alpha_{23}e_3+\alpha_{24}e_4+\alpha_{25}e_5+\alpha_{26}e_6+\alpha_{27}e_7+$$
$$\alpha_{23}(e_3+\alpha_{34}e_4+\alpha_{35}e_5+\alpha_{36}e_6+\alpha_{37}e_7)+\alpha_{24}(\alpha_{45}e_5+\alpha_{46}e_6+\alpha_{47}e_7)+\alpha_{25}(e_5+\alpha_{34}e_6+\alpha_{35}e_7)+\alpha_{26}\alpha_{45}e_7+\alpha_{27}e_7=$$
$$2\alpha_{23}e_3+(\alpha_{23}\alpha_{34}+\alpha_{24})e_4+(\alpha_{23}\alpha_{35}+\alpha_{24}\alpha_{45}+2\alpha_{25})e_5+(\alpha_{26}+\alpha_{23}\alpha_{36}+\alpha_{24}\alpha_{46}+\alpha_{25}\alpha_{34})e_6+(\alpha_{23}\alpha_{37}+\alpha_{24}\alpha_{47}+\alpha_{25}\alpha_{35}$$
$$+\alpha_{26}\alpha_{45}+2\alpha_{27})e_7$$

Tenglikning o‘ng tomoni uchun esa quyidagi tenglik o‘rinli bo‘ladi:

$$[[x_1,x_2,e_1],e_1,e_2]+[x_1,[e_1,x_2,e_1],e_2]+[x_1,e_1,[e_2,x_2,e_1]]=0+0+[x_1,e_1,\alpha_{23}e_3+\alpha_{24}e_4+\alpha_{25}e_5+\alpha_{26}e_6+\alpha_{27}e_7]=$$
$$\alpha_{23}(\alpha_{34}e_4+\alpha_{35}e_5+\alpha_{36}e_6+\alpha_{37}e_7)+\alpha_{24}(\alpha_{45}e_5+\alpha_{46}e_6+\alpha_{47}e_7)+\alpha_{25}(e_5+\alpha_{34}e_6+\alpha_{35}e_7)+\alpha_{26}(e_6+\alpha_{45}e_7)+2\alpha_{27}e_7=$$
$$\alpha_{23}\alpha_{34}e_4+(\alpha_{23}\alpha_{35}+\alpha_{24}\alpha_{45}+\alpha_{25})e_5+(\alpha_{23}\alpha_{36}+\alpha_{24}\alpha_{46}+\alpha_{25}\alpha_{34}+\alpha_{26})e_6+(\alpha_{23}\alpha_{37}+\alpha_{24}\alpha_{47}+\alpha_{25}\alpha_{35}+\alpha_{26}\alpha_{45}+2\alpha_{27})e_7$$

Yuqoridagi tengliklardan

$$\alpha_{23}=\alpha_{24}=\alpha_{25}=0$$

ekanligi kelib chiqadi.

$$[[x_1,e_1,e_2],x_3,e_1]=[[x_1,x_3,e_1],e_1,e_2]+[x_1,[e_1,x_3,e_1],e_2]+[x_1,e_1,[e_2,x_3,e_1]]$$

Tenglikning chap tomoni uchun quyidagi tenglik o‘rinli bo‘ladi:

$$[[x_1,e_1,e_2],x_3,e_1]=[e_2+\alpha_{23}e_3+\alpha_{24}e_4+\alpha_{25}e_5+\alpha_{26}e_6+\alpha_{27}e_7,x_3,e_1]=\alpha_{26}e_6+\alpha_{27}e_7+\alpha_{26}(e_6+\alpha_{45}e_7)=$$
$$2\alpha_{26}e_6+(\alpha_{27}+\alpha_{26}\alpha_{45})e_7$$

Tenglikning o‘ng tomoni uchun esa quyidagi tenglik o‘rinli bo‘ladi:

$[[x_1,x_3,e_1],e_1,e_2]+[x_1,[e_1,x_3,e_1],e_2]+[x_1,e_1,[e_2,x_3,e_1]]=[x_1,e_1,\alpha_{26}e_6+\alpha_{27}e_7]=\alpha_{26}(e_6+\alpha_{45}e_7)+2\alpha_{27}e_7=\alpha_{26}e_6+(\alpha_{26}\alpha_{45}+2\alpha_{27})e_7$

Yuqoridagi tengliklarni tenglashtirsak, u holda quyidagi natijalarni olamiz:

$$\alpha_{26}=\alpha_{27}=0$$

$$[[x_1,e_1,e_3],x_2,e_1]=[[x_1,x_2,e_1],e_1,e_3]+[x_1,[e_1,x_2,e_1],e_3]+[x_1,e_1,[e_3,x_2,e_1]]$$

Tenglikning chap tomoni uchun quyidagi tenglik o‘rinli bo‘ladi:

$[[x_1,e_1,e_3],x_2,e_1]=[\alpha_{34}e_4+\alpha_{35}e_5+\alpha_{36}e_6+\alpha_{37}e_7,x_2,e_1]=\alpha_{34}(\alpha_{45}e_5+\alpha_{46}e_6+\alpha_{47}e_7)+\alpha_{35}(e_5+\alpha_{34}e_6+\alpha_{35}e_7)+\alpha_{36}\alpha_{45}e_7+\alpha_{37}e_7=(\alpha_{34}\alpha_{45}+\alpha_{35})e_5+(\alpha_{34}\alpha_{46}+\alpha_{35}\alpha_{34})e_6+(\alpha_{34}\alpha_{47}+\alpha^2_{35}+\alpha_{36}\alpha_{45}+\alpha_{37})e_7$

Tenglikning o‘ng tomoni uchun esa quyidagi tenglik o‘rinli bo‘ladi:

$[[x_1,x_2,e_1],e_1,e_3]+[x_1,[e_1,x_2,e_1],e_3]+[x_1,e_1,[e_3,x_2,e_1]]=[x_1,e_1,e_3+\alpha_{34}e_4+\alpha_{35}e_5+\alpha_{36}e_6+\alpha_{37}e_7]=\alpha_{34}e_4+\alpha_{35}e_5+\alpha_{36}e_6+\alpha_{37}e_7+\alpha_{34}(\alpha_{45}e_5+\alpha_{46}e_6+\alpha_{47}e_7)+\alpha_{35}(e_5+\alpha_{34}e_6+\alpha_{35}e_7)+\alpha_{36}(e_6+\alpha_{45}e_7)+2\alpha_{37}e_7=\alpha_{34}e_4+(2\alpha_{35}+\alpha_{34}\alpha_{45})e_5+(2\alpha_{36}+\alpha_{34}\alpha_{46}+\alpha_{35}\alpha_{34})e_6+(3\alpha_{37}+\alpha_{34}\alpha_{47}+\alpha^2_{35}+\alpha_{36}\alpha_{45})e_7$

Yuqoridagi tengliklarni tenglashtirsak, u holda quyidagicha natijalarni olamiz:

$$\alpha_{34}=\alpha_{35}=\alpha_{36}=\alpha_{37}=0$$

$$[[x_1,e_1,e_4],x_2,e_1]=[[x_1,x_2,e_1],e_1,e_4]+[x_1,[e_1,x_2,e_1],e_4]+[x_1,e_1,[e_4,x_2,e_1]]$$

Tenglikning chap tomoni uchun quyidagi tenglik o‘rinli bo‘ladi:

$[[x_1,e_1,e_4],x_2,e_1]=[\alpha_{45}e_5+\alpha_{46}e_6+\alpha_{47}e_7,x_2,e_1]=\alpha_{45}e_5+\alpha_{46}\alpha_{45}e_7+\alpha_{47}e_7=\alpha_{45}e_5+(\alpha_{46}\alpha_{45}+\alpha_{47})e_7$

Tenglikning o‘ng tomoni uchun esa quyidagi tenglik o‘rinli bo‘ladi:

$[[x_1,x_2,e_1],e_1,e_4]+[x_1,[e_1,x_2,e_1],e_4]+[x_1,e_1,[e_4,x_2,e_1]]=[x_1,e_1,\alpha_{45}e_5+\alpha_{46}e_6+\alpha_{47}e_7]=\alpha_{45}e_5+\alpha_{46}(e_6+\alpha_{45}e_7)+2\alpha_{47}e_7=\alpha_{45}e_5+\alpha_{46}e_6+(\alpha_{46}\alpha_{45}+2\alpha_{47})e_7$

Yuqoridagi tengliklardan, $\alpha_{46}=\alpha_{47}=0$ ekanligi kelib chiqadi. ►

XULOSA

Ushbu maqolada maksimal hiponilpotent idealga ega 10-o‘lchamli yechiluvchan 3-Li algebrasi kengaytmasi o‘rganilgan. Kichik o‘lchamli maksimal hiponilpotent idealga ega 3-Li algebralarining tasnifini olishda berilgan algebraning differensiallashlar fazosi matritsaviy ko‘rinishi o‘rganildi va uni bazis almashtirishlar yordamida yuqori uchburchak shaklga keltirilib, mos chiziqli erkli differensiallashlar fazosiga ichki differensiallashlarni akslantirish yordamida bir nechta ko‘paytmalarni olamiz. Olingan ko‘paytmalarni Umumlashgan Yakobi ayniyatidan foydalanib ba’zi koeffitsiyentlar nolga tengligi olingan.

Foydalanilgan adabiyotlar:

1. K.K. Abdurasulov, R.K.Gaybullaev, B.A. Omirov, A.Kh. Khudoyberdiev Maximal Solvable Extension of Naturally Graded Filiform n-Lie Algebras. Sibirskii Matematicheskii Zhurnal, 2022, Vol. 63, No. 1, pp. 3-22.

2. Nambu Y., Generalized Hamiltonian dynamics, Phys. Rev., vol. 7, no. 8, 2405-2412 (1973).

3. Bai R., Shen C., and Zhang Y., Solvable 3-Lie algebras with a maximal hypo-nilpotent ideal, Electron. J. Linear Algebra,vol. 21, 43-62 (2010).

4. Ling W 1993 On the structure of n-Lie algebras, Dissertation University-GHS -Siegen, Siegn.

5. L.Takhtajan, On foundation of the generalized Nambu mechanics, Commun. Math. Phys., 1994, 160, 295-315.

6. R.P. Bai, J. Wang and Z. Li, Derivations of the n-Lie algebra realizated by J. Nonlinear Math. Phys. 18 (2011), no. 1, 151-160.

7. R.Bai, G.Song, Y.Zhang, The Classification of n-Lie Algebras, arXiv:1006.1932v1 [math-ph] 10 Jun 2010.

8. Y.Nambu, Generalized Hamiltonian Dynamics, Phys. Rev., 1973, D7, 2405-2412.

9. V. Filippov, n-Lie algebras, Sibirsk. Mat. Zh. 26 (1985), no. 6, 126-140, 191.

10. Jacobson N., A note on automorphisms and derivations of Lie algebras, Proc. Amer. Math. Soc., vol. 6, no. 2, 281-383(1955).

11. Bai R., Shen C., Zhang Y. *3-Lie algebras with an ideal N** Linear Algebra and its Applications, 2009.

12. Beshimova Sh.X., Gaybullayev R.K., Maximal extension of some sovable 3-Li algebras, Mathematics, machanics and intellectual technologies., 28-29 March 2023, Tashkent, Uzbekistan.