O'ZGARMAS KOEFFITSIYENTLI BIRJINSLI BO'LMAGAN CHIZIQLI TENGLAMALAR SISTEMASI FORMULALARI ISBOTI VA ULARGA DOIR MISOLLAR YECHISH

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KALIT SO'ZLAR:
Birjinslimas, sistema, tenglama, vektor,uzluksizlik O'zgarmas koeffisiyentli birjinsli bo'lmagan chiziqli tenglamalar sistemasi formulalari isboti va ularga doir misollar yechish

ANNOTATSIYA:

Ushbu maqolada biz o'zgarmas koeffisiyentli birjinsli bo'lmagan chiziqli tenglamalar sistemasi formulalarini va ularga doir misollar yechishning samarali usullaridan foydalanganmiz. Bu usullarning ketma-ket bajariladigan ikkita qismi bo'lib, ular quyidagi umumiy g'oyaga asoslanadi. Albatta, matematika alohida dunyoqarashga ega. Ma'lumki real obyektlar juda murakkab bo'ladi. Ularni o'rganish uchun modellar yasaladi. Modellarni o'rganish natijasida obyektlarga nisbatan xulosalar chiqariladi.

Ushbu tenglamalar sistemasiga

$$\begin{cases} \frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n + f_1(t) \\ \frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n + f_2(t) \\ .... \\ \frac{dx_n}{dt} = a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n + f_n(t) \end{cases} \quad (1.1)$$

yoki

$$\frac{dX}{dt} = AX + F(t) \qquad (1.1)$$

birinchi tartibli o'zgarmas koeffisiyentli chiziqli birjinslimas n-ta noma'lumli n-ta differensial tenglamalar sistemasi deyiladi.

Bu yerda

$F(t) = \{f_i(t)\} \quad (i = 1, n)$, $t \in [a, b]$ kesmada berilgan uzluksiz vektor funktsiyadir,

$a_{ki} - (k, i = 1,.., n)$ oldindan berilgan o'zgarmas sonlardir,

$A = \{a_{k_i}\}$ – kvadratik n-nchi tartibli matrisadir,

$X = \{x_i(t)\} \quad (i = 1,..., n)$ – izlanayotgan vektor-funktsiyadir.

$$F(t) = \begin{pmatrix} f_1(t) \\ f_1(t) \\ \vdots \\ f_n(t) \end{pmatrix}, \qquad X = \begin{pmatrix} x_1 \\ x_1 \\ \vdots \\ x_n \end{pmatrix}.$$

Faraz qilaylik tenglamalar sistemasi bir jinsli $F(t) = 0$ bo'lganda

$$\frac{dX}{dt} = A \cdot X$$

tenglamaning umumiy

$$\overline{X}(t) = \sum_{k=1}^{n} C_k X^{(k)}(t)$$

va birjinslimas (1.1) tenglamaning xususiy yechimi $X^*(t)$ bo'lsin.

U holda (1.1) tenglamaning umumiy yechimi:

$$X(t) = \overline{X}(t) + X^x(t)$$

yoki

$$X_i(t) = \sum_{k=1}^{n} C_k X^{(x)}(t) + X^*(t) \qquad (i = 1, n) \qquad (1.2)$$

Haqiqattan (1.2) yig'indi, C_k – doimiylar har qanday bo'lganda ham (1.1) ning yechimi bo'lganidan

$$L\left[\sum_{k=1}^{n} C_k X_i^{(n)}(t) + X^*\right] = L[X^*] = F(t)$$

Ikkinchi tomondan, $X(t)$ (1.1) ning yechimi bo'lsa

$$L[X - X^*] = L[X] - L[X^*] = F(t) - F(t) = 0,$$

U holda

$$X - X^* = \sum_{k=1}^{n} C_k X_i^{(k)}$$

Agar birjinsli $L[X] = 0$ tenglamaning umumiy yechimi ma’lum bo’lsa, birjinslimas tenglamaning yechimini ixtiyoriy o’zgarmaslarni variasiyalash (Lagranj usuli) bilan aniqlash mumkin.

Shunday qilib dastlab ko’rib o’tganimizdek birjinsli tenglama tuzamiz va bu tenglama uchun yuqoridagi usullardan birini qo’llab uning *n*-ta xususiy yechimini topamiz:

$$\left.\begin{matrix} \left[x_1^{(1)},\ x_2^{(1)},\ \ldots,\ x_n^{(1)},\right] \\ \left[x_1^{(2)},\ x_2^{(2)},\ \ldots,\ x_n^{(2)},\right] \\ \left[x_1^{(n)},\ x_2^{(n)},\ \ldots,\ x_n^{(n)},\right] \end{matrix}\right\}$$

Bu yerda

$$x_1^{(i)} = P_1^{(i)} e^{k_i(t)},\quad x_2^{(i)} = P_2^{(i)} e^{k_i(t)},\quad \ldots,\quad x_n^{(i)} = P_n^{(i)} e^{k_i(t)}.$$

Bu xususiy yechimlar fundamental sistemani tashkil qiladi.

Misol.

$$\begin{cases} \dfrac{dx_1}{dt} = x_1 + 2x_2 + e^t \\ \dfrac{dx_2}{dt} = 2x_1 + x_2 \end{cases}$$

sistema yechilsin.

1. Dastlab birjinsli tenglamani qaraymiz.

$$\begin{cases} \dot{x}_1(t) = x_1 + 2x_2 \\ \dot{x}_2(t) = x_1 + x_2 \end{cases} \qquad (A)$$

Bu sistema uchun xarakteristik tenglama quyidagicha bo’ladi.

$$\begin{vmatrix} 1-k & 2 \\ 2 & 1-k \end{vmatrix} = 0$$

$$(1-k)^2 - 4 = 0,\qquad 1 - 2k + k^2 - 4 = 0,\qquad k^2 - 2k - 3 = 0,$$

$$k_{1,2} = 1 \pm \sqrt{1+3} = 1 \pm 2,\qquad k_1 = 1 - 2 = -1,\qquad k_2 = 1 + 2 = 3$$

Xarakteristik tenglamaning ildizlari haqiqiy va har xil? demak

$$x_1(t) = c_1 e^{-t} + c_2 e^{3t}$$
$$x_2(t) = a_1 e^{-t} + a_2 e^{3t}$$

Bu yerdan:

$$\dot{x}_1(t) = -c_1 e^{-t} + 3c_2 e^{3t},$$
$$\dot{x}_2(t) = -a_1 e^{-t} + 3a_2 e^{3t}.$$

endi $x_1(t),\quad x_2(t)$ **va** $\dot{x}_1(t),\quad \dot{x}_2(t)$ **larning ifodalarini (A) sistemaga qo'yamiz:**

$$\begin{cases} -c_1 e^{-t} + 3c_2 e^{3t} = c_1 e^{-t} + c_2 e^{3t} + 2c_1 e^{-t} + 2a_2 e^{3t} \\ -a_1 e^{-t} + 3a_2 e^{3t} = +2c_1 e^{-t} + 2c_2 e^{3t} + a_1 e^{-t} + 1a_2 e^{3t} \end{cases}$$

$$\begin{cases} -2c_1 e^{-t} + 2c_2 e^{3t} = +2a_1 e^{-t} + 2a_2 e^{3t} \\ 2c_1 e^{-t} + 2c_2 e^{3t} = a - 2ae^{-t} + 2a_2 e^{3t} \end{cases}$$

$$\begin{cases} -c_1 e^{-t} + c_2 e^{3t} = +a_1 e^{-t} + a_2 e^{3t}, \\ c_1 e^{-t} + c_2 e^{3t} = 0 - a_1 e^{-t} + a_2 e^{3t} \end{cases} \qquad \begin{cases} a_1 = -c_1 \\ a_2 = c_2 \end{cases}$$

Demak birjinsli tenglamaning umumiy yechimi:

$$\left.\begin{aligned} x_1(t) &= c_1 e^{-t} + c_2 e^{3t} \\ x_2(t) &= -c_1 e^{-t} + c_2 e^{3t} \end{aligned}\right\} \qquad (B)$$

2. Endi birjinslimas tenglamaning yechimini aniqlaymiz. Buning uchun

$$c_1 = c_1(t), \qquad c_2 = c_2(t)$$

deb qaraymiz. Lagranj usulini qo'llaymiz. (1.6) ga ko'ra:

$$\begin{cases} c_1'(t) e^{-t} + c_2'(t) e^{3t} = e^{t}, \\ -c_1'(t) e^{-t} + c_2'(t) e^{3t} = 0. \end{cases}$$

Bu sistemaning determinanti Vronskiondir:

$$W = \begin{vmatrix} e-t & e^{3-6} \\ -e^{-t} & e^{3t} \end{vmatrix} = e^{-t} \cdot e^{3t} \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = e^{2t} \cdot (1+1) = 2e^{2t} \neq 0$$

Shu sabab sistema yechimga ega.

Hadma-had qo'shamiz:

$$2c_2' e^{3t} = e^{t}, \qquad 2c_2' = e^{-2t}, \qquad c_2' = \frac{1}{2} e^{-2t},$$

$$c_2(t) = \frac{1}{2} \int e^{-2t} dt + \bar{c}_2 = -\frac{1}{4} e^{-2t} + \bar{c}_2$$

Hadma-had ayiramiz:

$$2c_1{}' e^{-t} = e^t, \quad 2c_1{}' = e^{2t} \quad c_1{}' = \frac{1}{2}e^{2t},$$

$$C_1(t) = \frac{1}{2}\int e^{2t}dt = \frac{1}{4}e^{2t} + \bar{c}_1$$

Shunday qilib, umumiy yechim ushbu ko'rinishda bo'ladi:

$$\left.\begin{aligned} x_1(t) &= \left(\frac{1}{4}e^{2t} + \bar{c}_1\right)\cdot e^{-t} + \left(-\frac{1}{4}e^{-2t} + \bar{c}_2\right)\cdot e^{3t} \\ x_2(t) &= -\left(\frac{1}{4}e^{2t} + \bar{c}_1\right)\cdot e^{-t} + \left(-\frac{1}{4}e^{-2t} + \bar{c}_2\right)\cdot e^{3t} \end{aligned}\right\}$$

yoki

$$\left.\begin{aligned} x_1(t) &= \bar{c}_1 e^{-t} + \bar{c}_2 e^{3t}, \\ x_2(t) &= -\bar{c}_1 e^{-t} + \bar{c}_2 e^{3t} - \frac{1}{2}e^t. \end{aligned}\right\}$$

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