

**MATHEMATICAL MODELING OF IMC TROLLEYBUS OPERATION ON THE
URGANCH–KHIVA ROUTE**

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**ARTICLE
INFORMATION**

ANNOTATION

**ARTICLE
HISTORY:**

Received: 16.03.2026

Revised: 17.03.2026

Accepted: 18.03.2026

KEYWORDS:

*IMC trolleybus,
mathematical model,
energy consumption,
traction force, SOC,
intercity transport,
regenerative braking.*

In this article, a mathematical model of an IMC (In-Motion Charging) trolleybus for the new route between the cities of Urganch and Khiva has been developed. The study analyzes the vehicle's operating conditions, traction forces, motor power, and battery charge. The model demonstrates the energy efficiency of the IMC system, the possibility of optimizing battery capacity, and energy savings through regenerative braking. The results of the study provide a basis for developing scientific recommendations for implementing intercity IMC systems in the conditions of Uzbekistan.

Introduction. The Role of IMC Trolleybus Technology in Intercity Transport

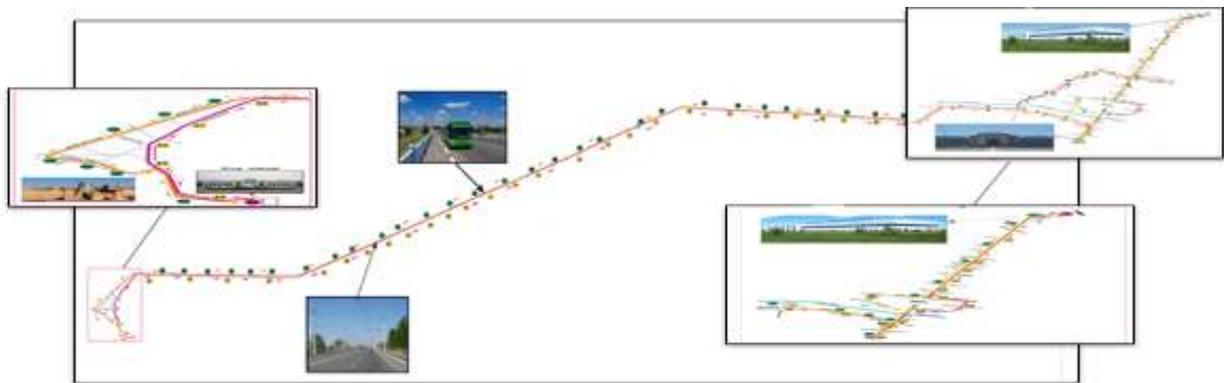
The development of electric transport systems enables increased stability of intercity routes and the implementation of environmentally friendly and energy-efficient transportation. In particular, trolleybuses equipped with IMC (In-Motion Charging) technology can charge their batteries while in motion, ensuring uninterrupted operation on wireless segments. This system allows for a reduction in battery mass, optimization of operational costs, and increased opportunities for energy savings.

The new route between the cities of Urganch and Khiva has been selected as a convenient experimental object for implementing the IMC system. The total route length is 72.4 km with 100 stops. Along the route, wireless segments include the sections from the Airport to Mash'al, from the last stop in Khiva to Khiva station, and the return segment from Mash'al to the Airport. The route is mostly flat, which provides favorable conditions for enhancing the energy efficiency of the IMC system and optimizing battery performance.

Segments of the Urganch–Khiva IMC trolleybus route

Table 1

No	Route Segment	Distance (km)
1	Total length of the new route	72.5
2	Airport → Mash'al (wireless segment)	8.0
3	Mash'al → Last stop in Khiva (wired segment)	27.2
4	Last stop in Khiva → Khiva station (wireless segment)	1.05
5	Khiva station → Last stop in Khiva (wireless segment)	1.05
6	Last stop in Khiva → Mash'al (wired segment)	27.2
7	Mash'al → Airport (wireless segment)	8.0



(Figure 1) Route Optimized Based on In-Motion Charging

The objective of this study is to mathematically model the operation of an IMC trolleybus under the conditions of this route, analyze its energy consumption and battery charge status, and identify opportunities for energy savings through regenerative braking and on wireless segments. This model provides a basis for developing scientifically grounded recommendations for implementing intercity IMC systems in the conditions of Uzbekistan.

An important part of the route is the segment integrated with Urganch International Airport. Due to technical and aesthetic constraints on installing the contact network within the airport area, certain sections are operated in wireless mode. This approach allows the transport system to be integrated with the airport, railway station, and city center, enabling the development of a multimodal transport system.

Mathematical model: physical and energetic principles of imc trolleybus operation. Vehicle dynamics: traction forces, aerodynamic and wheel resistance, terrain effects, and inertia.

1. F_{roll} – rolling friction- In this way, the wheel in contact experiences a constant resistance.

$$F_{roll} = C_r m g$$

C_r - Wheel rolling resistance coefficient (usually 0.005–0.015)

2. F_{grade} – gradient resistance .

$$F_{\text{grade}} = m g \sin \theta$$

(The gradient on the Urganch – Khiva route is almost flat $\theta \approx 0$)

3. F_{aero} – Aerodynamic drag.

$$F_{\text{aero}} = \frac{1}{2} \rho A_f C_d v^2$$

ρ - Air density, A_f -frontal area, C_d -aerodynamic coefficient, v -vehicle speed.

4. F_{inertia} – Inertia resistance

$$F_{\text{inertia}} = ma$$

This is the required force when the trolleybus is accelerating or decelerating.

5. F_{total} –The sum of all forces gives the total resistance:

$$F_{\text{total}} = F_{\text{roll}} + F_{\text{inertia}} + F_{\text{aero}} + F_{\text{grade}}$$

Wheel Torque Calculation The torque transmitted to the wheel:

$$T_{\text{wheel}} = \frac{F_{\text{total}} R_{\text{wheel}}}{\eta_{\text{transmissiya}}}$$

where: R_{wheel} – wheel radius, $\eta_{\text{transmissiya}}$ – transmission efficiency (typically 0.95–0.98).

Wheel angular velocity

The wheel angular velocity (rad/s) is calculated from vehicle speed:

$$\omega_{\text{wheel}} = \frac{v}{R_{\text{wheel}}}$$

Transmission system and motor parameters.

The torque generated at the trolleybus wheel is transmitted to the electric motor shaft through the transmission system, accounting for mechanical losses and inertia.

Torque transformation. Torque at the motor shaft:

$$T_{\text{motor}} = \frac{T_{\text{wheel}}}{i_{\text{gear}} \eta_{\text{gear}}}$$

i_{gear} –gear ratio, η_{gear} –mechanical efficiency.

Total required torque. The total torque acting on the motor shaft:

$$T_{\text{motor required}} = T_{\text{motor}} + T_{\text{idling}} + T_{\text{inertia}}$$

Idling resistance torque:

$$T_{\text{idling}} = \text{const}$$

Inertia torque:

$$T_{\text{inertia}} = J \frac{d\omega_{\text{motor}}}{dt}$$

where J is the equivalent moment of inertia.

Motor angular velocity transformation

$$\omega_{\text{motor}} = \omega_{\text{wheel}} i_{\text{gear}}$$

Induction motor power. The electric motor operation is determined by torque and speed, bounded by physical limits:

$$T_{\text{min}} \leq T_{\text{motor}} \leq T_{\text{max}}, \quad 0 \leq \omega_{\text{motor}} \leq \omega_{\text{max}}$$

Mechanical power at motor shaft:

$$P_{\text{mech}} = T_{\text{motor}} \cdot \omega_{\text{motor}}$$

Traction Mode ($T_{\text{motor}} \geq 0$)

Electric energy is converted to mechanical energy:

$$P_{\text{motor_traction}} = \frac{T_{\text{motor}} \cdot \omega_{\text{motor}}}{\eta} \quad \eta \text{— motor efficiency}$$

Regeneration mode ($T_{\text{motor}} < 0$)

Motor operates as a generator, converting kinetic energy to electric energy:

$$P_{\text{regen}} = T_{\text{motor}} \cdot \omega_{\text{motor}} \cdot \eta_{\text{regen}}$$

where η_{regen} — regeneration efficiency.

Total motor power

$$P_{\text{motor}} \begin{cases} \frac{T_{\text{motor}} \cdot \omega_{\text{motor}}}{\eta}, & (T_{\text{motor}} \geq 0) \\ T_{\text{motor}} \cdot \omega_{\text{motor}} \cdot \eta_{\text{regen}}, & (T_{\text{motor}} < 0) \end{cases}$$

Wired sections representation.

The trolleybus position along the route:

$$s = s_{\text{bus}}(t)$$

Where: s — distance traveled (m), t — time..

Wired intervals along the route:

1- Outbound wired section: $8000 \leq s_{\text{bus}} < 35300$

2- Return wired section: $37400 \leq s_{\text{bus}} < 64700$

Wired section indicator function:

$$\text{Interval1}(s) \begin{cases} 1, & 8000 \leq s_{\text{bus}} < 35300 \\ 0, & \text{otherwise} \end{cases}$$

$$\text{Interval2}(s) \begin{cases} 1, & 37400 \leq s_{\text{bus}} < 64700 \\ 0, & \text{otherwise} \end{cases}$$

Overall wired indicator:

A logical function that indicates whether the trolleybus is currently in a wired section or a wireless section:

$$isWired(s) = \text{Interval1}(s) \vee \text{Interval2}(s)$$

Mathematically, this can be written as follows:

$$isWired \begin{cases} 1, & 8000 \leq s_bus < 35300 \\ 1, & 37400 \leq s_bus < 64700 \\ 0, & otherwise \end{cases}$$

where:

isWired=1 → the trolleybus is connected to the contact network

isWired=0 → the trolleybus operates in wireless (battery) mode

Grid power limitation.

The maximum power that can be drawn from the electrical grid is determined as follows:

$$P_{grid,max}(s) = isWired(s) \cdot P_{grid}^{val}$$

where:

$$P_{grid}^{val} = 160\,000\,W$$

$$P_{grid,max}(s) = \begin{cases} 160\,000, & isWired = 1 \\ 0, & isWired = 0 \end{cases}$$

Thus:

In the wired section → the maximum available power is 160 kW.

In the wireless section → the grid power is equal to zero.

Mathematical representation of the energy mode.

Total power balance of the trolleybus:

$$P_{req}(t) = P_{grid}(t) + P_{bat}(t)$$

Constraint condition:

$$0 \leq P_{grid}(t) \leq P_{grid,max}(s)$$

If:

isWired = 1 → the main power is supplied from the grid, and excess energy charges the battery.

isWired = 0 → the entire required power is supplied from the battery.

Distribution of electrical energy between the grid and the battery

The total power demand of the trolleybus is formed by the sum of the traction motor power and the auxiliary systems power:

$$P_{total} = P_{motor} + P_{aux}$$

where:

P_{motor} -traction motor power,

P_{aux} - power of auxiliary systems (air conditioning, lighting, control systems, etc.).

The maximum power that can be supplied by the electrical grid is limited by:

$$P_{grid} \leq P_{grid}^{max}$$

It is defined by the following limit (constraint).

Power required from the battery.

If the total power demand exceeds the capability of the electrical grid, the missing power is supplied by the battery:

$$P_{bat}^{req} = \begin{cases} P_{total} - P_{grid}^{max}, & P_{total} > P_{grid}^{max} \\ 0, & P_{total} \leq P_{grid}^{max} \end{cases}$$

Battery Charging Power

If the power available from the electrical grid exceeds the required demand, the excess energy is directed to charge the battery:

$$P_{bat}^{req} = \begin{cases} P_{grid}^{max} - P_{total}, & P_{grid}^{max} > P_{total} \\ 0, & P_{grid}^{max} \leq P_{total} \end{cases}$$

Energy balance. The overall power balance of the system can be expressed as:

$$P_{motor} + P_{aux} = P_{grid} + P_{bat}$$

where:

$$P_{bat} \begin{cases} P_{bat}^{req}, & \text{battery discharging} \\ P_{bat}^{chg}, & \text{battery charging} \end{cases} \quad P_{grid} \leq P_{grid}^{max}$$

Determination of battery state of charge (SOC).

The battery state of charge changes over time depending on the energy balance of the system. The net power is determined based on the power entering and leaving the battery.

Battery power balance. The net battery power is defined as:

$$P_{net} = P_{bat}^{req} - P_{regen} - P_{bat}^{chg}$$

P_{bat}^{req} – power drawn from the battery,

P_{regen} – power obtained from regenerative braking,

P_{bat}^{chg} – power supplied to the battery from excess grid energy.

If $P_{net} > 0$ the battery is **discharging**,

If $P_{net} < 0$ the battery is **charging**.

Charging efficiency. Energy losses during the charging process are considered as:

$$P_{chg}^{eff} = \eta_{chg} P_{chg}$$

where: $\eta_{chg} \approx 0.95$

Discharge efficiency. The actual power delivered from the battery during discharge is expressed as:

$$P_{dis}^{eff} = P_{chg}^{eff} - P_{dis}^{eff}$$

Time variation of SOC. Battery energy is calculated as:

$$E_{bat} = \frac{P_{bat}^{eff}}{3600}$$

Agar batareyaning umumiy sig‘imi E_{cap} bo‘lsa, SOC ning o‘zgarish tezligi:

$$SOC = \frac{P_{bat}^{eff}}{3600E_{cap}}$$

SOC calculation. SOC is obtained by integrating over time:

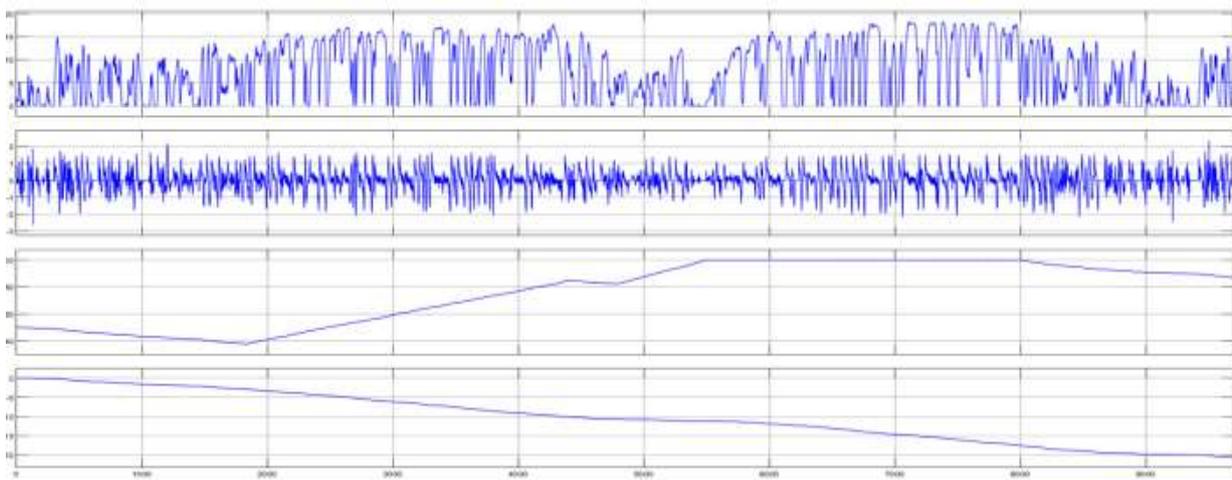
$$SOC(t) = SOC_0 + \int SOC dt$$

The result is expressed as a percentage:

$$SOC_{\%} = 100 \cdot SOC$$

and remains within the following limits: $0 \leq SOC \leq 1$

Calculation results and energy analysis.



(Figure 2) Graphs of velocity, acceleration, and energy variations in the MATLAB/SIMULINK model

In determining the traction battery capacity, we based the calculation on the initial wireless segment, which represents the most energy-demanding operating condition. This approach corresponds to the “calculation based on the critical operating mode” method commonly used in the design of electric vehicles, where the maximum energy consumption on the segment where the battery operates independently is the decisive factor in selecting the battery capacity.

Steps for determining traction battery capacity for the imc trolleybus:

Selected segment: Segment 1 – Airport to Mash’al

Distance: 8 km

Net energy consumption: 15.79 kWh (the highest among all wireless segments)

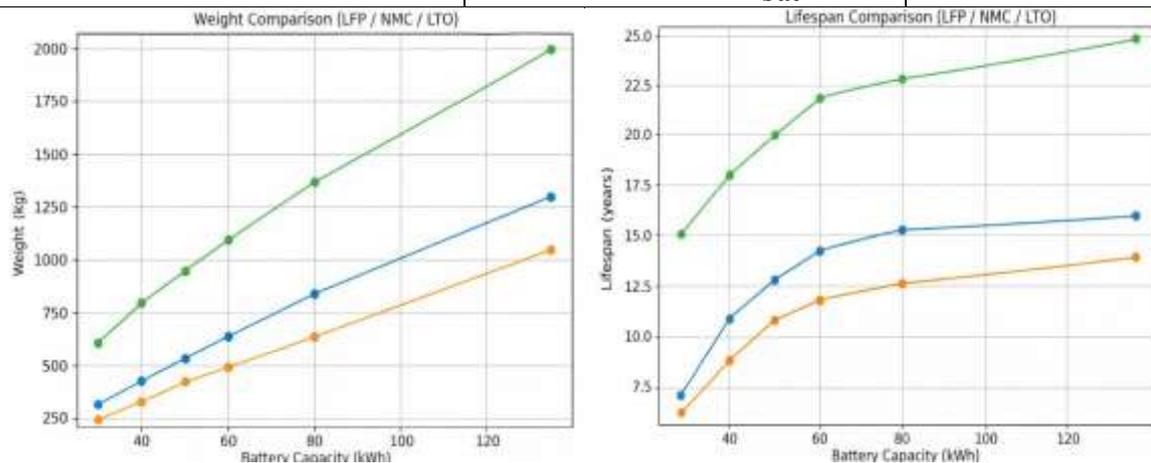
Travel time: 1836 s ≈ 0.51 h

Energy consumption per kilometer:

Energy per km=8 km/15.79 kWh=1.97 kWh/km

Table 2

Step	Parameter	Designation / Formula	Value
1	Energy demand of the most demanding wireless segment	E_{need}	15.79 kWh
2	Travel time	(t)	1836 s (0.51 soat)
3	Average power	$P_{avg} = \frac{E}{t}$	≈ 31 kW
4	Battery efficiency	η_{bat}	0.92
5	Allowed depth of discharge	(DoD)	0.8
6	Nominal capacity (calculated)	$C_{nom} = \frac{E_{need}}{\eta_{bat} \cdot DoD}$	21.45 kWh
7	Operational reserve (margin)	—	15 %
8	Final nominal capacity	$C_{final} = C_{nom}(1 + margin)$	24.66 kWh
9	Practically selected capacity	—	≈ 40 kWh
10	Average C-rate	$C - rate = \frac{P_{avg}}{E_{bat}}$	≈ 0.78 C



Comparison of capacity and cost for different battery technologies.

Table 3

Capacity (kWh)	LFP Price (\$/kWh)	LFP Total (\$)	NMC Price (\$/kWh)	NMC Total (\$)	LTO Price (\$/kWh)	LTO Total (\$)
40	~85	~3,400	~115	~4,600	~175	~7,000
60	~85	~5,100	~115	~6,900	~175	~10,500
80	~85	~6,800	~115	~9,200	~175	~14,000
100	~85	~8,500	~115	~11,500	~175	~17,500
120	~85	~10,200	~115	~13,800	~175	~21,000
127	~85	~10,795	~115	~14,605	~175	~22,225

Conclusion.

In this study, the movement of the IMC (In-Motion Charging) trolleybus along the Urganch–Khiva route was mathematically analyzed. The main forces affecting the trolleybus motion, the power demand of the traction motor, and energy consumption were taken into account, and the energy distribution along wired and wireless segments of the route was evaluated. Using the model, variations in battery state of charge and energy recovery through regenerative braking were analyzed.

According to the calculation results, the wireless segment with the highest energy demand was taken as the primary criterion for determining the traction battery capacity. Considering energy consumption, battery efficiency, and the allowable depth of discharge, the minimal required traction battery capacity was determined, and a practical value with operational reserve was established for real use.

Additionally, batteries of different capacities were compared from technological and economic perspectives, evaluating their cost, service life, and operational characteristics.

The obtained results can be used for the practical implementation of IMC trolleybus systems on intercity routes, selecting battery parameters, and optimizing energy management.

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